1. Wave function **(4 pts)**

Show that eqn. (2.64), which describes a plane wave of arbitrary form, satisfies the wave equation.

$$\frac{\partial^2\Psi}{\partial x^2} = \alpha^2 f''; \frac{\partial^2\Psi}{\partial y^2} = \beta^2 f''; \frac{\partial^2\Psi}{\partial z^2} = \gamma^2 f''; \frac{\partial^2\Psi}{\partial t^2} = v^2 f''$$

$\Delta\Psi - \frac{1}{v^2}\frac{\partial^2\Psi}{\partial t^2} = \left(\alpha^2 + \beta^2 + \gamma^2 - 1\right) f'' = 0$ whenever $\alpha^2 + \beta^2 + \gamma^2 = 1$.

2. Photon flux **(4 pts)**

How many photons per second are emitted from a 100 W yellow lightbulb (average wavelength, λ = 550 nm) if 2.5% of the applied energy is emitted as light? (The remainder is just dissipated a heat.)

$P/(h\nu) = P\lambda/(hc) = 0.025 \cdot 100$ J/s $\cdot$ 550 nm/($6.63\cdot10^{-34}$ Js $\cdot$ $3\cdot10^8$ m/s) = $6.9\cdot10^{18}$ s^{-1}

3. Dielectric medium **(2 pts)**

Determine the index of refraction of a medium that reduces the speed of light by 10% as compared to its speed in vacuum.

$n = c/v = c/(0.9\cdot c) = 1.11$

4. Inversion of the dispersion equation **(4 pts)**

Show that the dispersion eqn. (3.70) can be rewritten as $1/(n^2 - 1) = -C/\lambda^2 + C/\lambda_0^2$ and determine C.

$n^2 = 1 + C^*/(\omega_0^2 - \omega^2)$ with $C^* = Nq_e^2/(\varepsilon_0\cdot m_e)$

$\rightarrow 1/(n^2 - 1) = (\omega_0^2 - \omega^2)/C^* = \frac{4\pi^2 c^2}{C^*}\left(\frac{1}{\lambda_0^2} - \frac{1}{\lambda^2}\right) = C\left(\frac{1}{\lambda_0^2} - \frac{1}{\lambda^2}\right)$

$\rightarrow C = \frac{4\pi^2 c^2 \varepsilon_0 m_e}{N q_e^2}$

5. Application of the Cauchy equation **(4 pts)**

Crystal quartz has refractive indices of $n_R = 1.557$ at $\lambda = 410.0$ nm and $n_R = 1.547$ at 550.0 nm. Using

$n_R \approx C_0 + C_1/\lambda^2 + O(\lambda^{-4})$, the Cauchy approximation, estimate n_R at 610.0 nm.

$n_{R,i} = C_0 + C_1/\lambda_i^2 \rightarrow \Delta n = n_{R,1} - n_{R,2} = C_1(\lambda_1^{-2} - \lambda_2^{-2})$ and $C_1 = \Delta n_R \cdot \lambda_1^2 \cdot \lambda_2^2 /(\lambda_2^2 - \lambda_1^2) = 3.78 \cdot 10^3$ nm^2.

Then, $n_{R,3} = n_{R,1} - C_1(\lambda_1^{-2} - \lambda_3^{-2}) = 1.557 - 0.012 = 1.545$

6. CO dissociation **(2 pts)**

If a photon is to dissociate a CO molecule into oxygen and carbon atoms, it must provide 11 eV of energy. What is the frequency of this UV photon?

$\nu = E/h = 11 \cdot 1.6 \cdot 10^{-19}$ J / $(6.63 \cdot 10^{-34}$ Js$) = 2.65 \cdot 10^{15}$ Hz